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a three-phase power supply system
in an electric vehicle charging station

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REACTIVE POWER COMPENSATION IN A THREE-PHASE POWER SUPPLY SYSTEM IN AN ELECTRIC VEHICLE CHARGING STATION

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Abstract: The article presents the problem of reactive power compensation in an electric network, which powers a charging station for electric vehicles. The reactive power is present because the converters have a reactance character. Mobile inverters are single-phase receivers, so they cause a lack of symmetry in a three-phase system. Lack of symmetry causes an unbalance component to appear. The power compensation discussed here results in an improvement in the power factor. The reduction of the reactive and unbalance power is the result of this compensation. The article takes into account a non-sinusoidal method of power supply.

Keywords: power compensation, reactive power, three-phase circuits, unbalanced system, nonlinear receiver

1. INTRODUCTION

Electric vehicle charging stations use several standards of plugs and voltages. AC and DC systems can be found here.

Fig. 1. Fast 150 kW charging station for an electric vehicle [17]

There are several dozen models of Electric Vehicle (EV) already available on the market. Different models have different types of charging plugs. Most plug-in standards on the markets use alternating current (AC) because this is easily available in every household. The use of alternating current has forced car manufacturers to use internal chargers that convert AC to DC current.

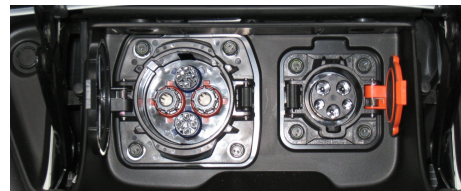Fig. 2. CHAdeMO socket (on the left) next to the Type 1 socket (on the right) [18]

The CHAdeMO standard allows for the delivery of direct current up to 62.5 kilowatts (500 volts, 125 amps). The CHAdeMO standard is designated IEC 62196 Type 4, but it is rarely called Type 4.

Type 2 is marked with IEC 62196 Type 2 code and allows charging with both current AC and DC.

Fig. 3. Type 2 / Mennekes Standard (AC / DC) [18]

Combined Charging System (CCS) Combo 2 is an extension of the Type 2 standard with two large DC charging pins. The pins responsible for the supply of alternating current have disappeared from the Type 2 plug.
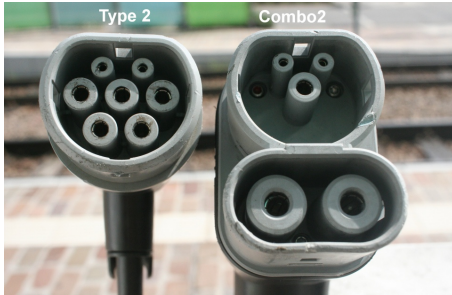

Fig. 4. Type 2 plug (on the left) next to the plug (right) Type 2 CCS / Type 2 combo / Combo 2 [18]

Type 1 (Fig. 2) allows charging with capacities up to 19.2 kW. This plug only supplies alternating current (AC), one or two phases.

Type 1 combo is an extension of the Type 1 plug with two thick pins for DC charging.

Fig. 5. Type 1 combo / Combo1 / CCS combo 1 / Type 1 CCS (DC) [18]

There are many other plug-in standards that are dedicated to selected car models. The examples of standards presented here show that charging stations for electric vehicles operate asymmetrically.
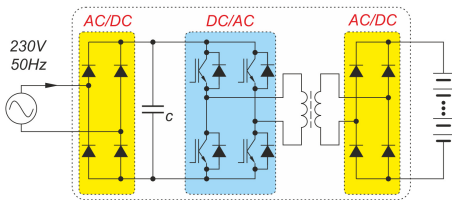

Fig. 6. Block diagram of the voltage converter

The generation of direct current from the three-phase alternating current supplying the station takes place in three-phase rectifiers. This means an even load on all the phases. The use of a single-phase AC charging socket results in an uneven distribution of currents in the supply network phases (Fig. 7).
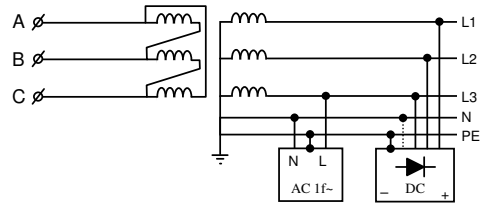

Fig. 7. Connection of AC and DC outlets in the charging station

The compensator used in the charging station must take into account the unbalance, reactance nature and nonlinearity of the receiver. The receiver's nonlinearity effect can be reduced by using a filter tuned to the appropriate harmonic. In the further part of this work, the nonlinearity problem was omitted.

In the further part of the work, the least favorable situation was assumed, in which only one phase is burdened.

2. AMBIGUITY IN THE INTERPRETATION OF APPARENT AND REACTIVE POWER

Determining the compensator parameters is an issue that requires understanding the energy properties of the electrical system. The method of describing the power in the system is a key issue in this problem.

Active power for the passive system has been precisely described by the active component of the current according to the Fryze's theory [2]. The definition of apparent power in three-phase circuits is not unambiguous.

In 1922 F.Bucholz proposed the definition of apparent power for a three-phase source with sinusoidal waveforms:

$$S = \sqrt{U_A^2 + U_B^2 + U_C^2} \cdot \sqrt{I_A^2 + I_B^2 + I_C^2} \ . \tag{1}$$

The apparent power presented in this form has not been accepted in technology but, in special cases it is still used today. Apparent power presented in 1935 by H.L. Curtis and F.B. Silsbee known as arithmetic apparent power became more widespread:

$$\underline{S} = \underline{U}_A \underline{I}_A^* + \underline{U}_B \underline{I}_B^* + \underline{U}_C \underline{I}_C^* = S \cdot e^{j\varphi} \ . \tag{2}$$

The third definition of this power is the form often used, known as geometrical apparent power:

$$S = \sqrt{P^2 + Q^2} \ . \tag{3}$$

Definitions (1), (2) and (3) give the same results only if: there is symmetry of the power source, the three-phase receiver is balanced, the current and voltage have the first harmonic only. When even one of these conditions is not met, the results obtained from these equations are contradictory.

Correct implementation of power compensation, and determination of energy properties of three-phase circuits is therefore related to the accepted definition of apparent power. Building power theory on several foundations of apparent power causes that fact that the obtained results may be contradictory even in the situation of sinusoidal waveforms. Assuming the power equation, analogously to single-phase, linear, sinusoidal systems, it should be stated that only the geometric apparent power definition (3) is correct in this case.

The power factor is closely related to the efficiency in the energy system. The reduction of the power factor is caused by an increase of transmission losses. Considering this, Czarnecki proposes to choose the definition of apparent power, paying particular attention to the assessment of these losses. After analyzing the unbalanced circuit, he stated that only the apparent power calculated according to Equation (1) gives a correct value of the power factor. This definition is an extension of the definitions $S = \|u\| \cdot \|i\|$ for single-phase circuits.

In the literature, the definition of reactive power is also unambiguous. However, looking at this power as a parameter directly affecting the selection of the compensator, one must accept the definition of this power in accordance with the CPC (Currents' Physical Components) theory.

3. DECOMPOSITION OF CURRENT INTO COMPONENTS

The method of decomposition of current into components according to the CPC theory [1] gives the best view on ways of decreasing the effective value of current. Knowledge of the properties of individual components of the current makes an analysis of the circuit in terms of the possibility of reactive power compensation easier.

Assuming the definitions of three-phase current and voltage vectors in the case of periodic runs for the n-th harmonic, it is possible to extend the sum of the harmonics of voltages and currents:

$$u = \sum_n u_{(n)} = \sqrt{2} \cdot \Re e \sum_n \underline{U}_{(n)} \cdot e^{jn\omega_1 t}, \tag{1}$$

$$u_{(n)} = \begin{bmatrix} u_{A(n)} \\ u_{B(n)} \\ u_{C(n)} \end{bmatrix}, \tag{2}$$

$$i = \sum_n i_{(n)} = \sqrt{2} \cdot \Re e \sum_n \underline{I}_{(n)} \cdot e^{jn\omega_1 t}, \tag{3}$$

$$i_{(n)} = \begin{bmatrix} i_{A(n)} \\ i_{B(n)} \\ i_{C(n)} \end{bmatrix}, \tag{4}$$

where the relevant complex vectors take the following form:

$$\underline{U}_{(n)} = \begin{bmatrix} \underline{U}_{A(n)} \\ \underline{U}_{B(n)} \\ \underline{U}_{C(n)} \end{bmatrix}, \tag{5}$$

$$\underline{I}_{(n)} = \begin{bmatrix} \underline{I}_{A(n)} \\ \underline{I}_{B(n)} \\ \underline{I}_{C(n)} \end{bmatrix}. \tag{6}$$

The effective electric energy of the n-th harmonic transmitted from the source to the three-phase receiver will be equal to the sum of energy for each phase separately.

$$P_{(n)} = \frac{1}{T} \int_0^T \left(u_{A(n)} i_{A(n)} + u_{B(n)} i_{B(n)} + u_{C(n)} i_{C(n)} \right) dt, \tag{10}$$

If the active power flowing for each harmonic is unidirectional, the resultant total active power is equal to:

$$P = \frac{1}{T} \int_0^T u^T i \, dt = \Re e \sum_n \underline{U}_{(n)}^T \underline{I}_{(n)}^*. \tag{11}$$

The active power of a three-phase system is connected with the equivalent admittance, defined for the n-th harmonic:

$$\underline{Y}_{e(n)} = G_{e(n)} + jB_{e(n)} = \underline{Y}_{AB(n)} + \underline{Y}_{BC(n)} + \underline{Y}_{CA(n)}. \tag{12}$$

The unbalance of a three-phase system translates into a non-zero value of unbalance admittance. For harmonics, where the voltages are in correct sequence, the unbalance admittance equals to:

$$\underline{A}_{(n)} = -\left(\underline{Y}_{BC(n)} + \alpha \underline{Y}_{CA(n)} + \alpha^* \underline{Y}_{AB(n)} \right) = $$
$$= A_{(n)} e^{j\psi_{(n)}}, \qquad \text{for } n = 3k+1, \tag{13}$$

where α - a complex rotation coefficient that is equal to: $\alpha = 1e^{j\frac{2\pi}{3}}$, $\alpha^* = 1e^{-j\frac{2\pi}{3}}$.

For these harmonics, the voltage vector (5) taking into account the rotation coefficient α is equal to:

$$\underline{\boldsymbol{U}}_{(n)}^{\#} = \begin{bmatrix} \underline{U}_{A(n)} \\ \alpha^{*} \cdot \underline{U}_{B(n)} \\ \alpha \cdot \underline{U}_{C(n)} \end{bmatrix} = \begin{bmatrix} \underline{U}_{A(n)} \\ \underline{U}_{C(n)} \\ \underline{U}_{B(n)} \end{bmatrix}, \quad \text{for} \quad n = 3k+1. \quad (7)$$

Therefore, when the voltages are in a correct sequence for the *n*-th harmonic ($n = 3k + 1$), a current-voltage relationship occurs:

$$\underline{\boldsymbol{I}}_{(n)} = \begin{bmatrix} \underline{Y}_{e(n)} + \underline{A}_{(n)} \\ \underline{Y}_{e(n)} + \alpha^{*} \cdot \underline{A}_{(n)} \\ \underline{Y}_{e(n)} + \alpha \cdot \underline{A}_{(n)} \end{bmatrix} \cdot \underline{\boldsymbol{U}}_{(n)}. \quad (8)$$

For harmonics where the voltages are in an incorrect sequence ($n = 3k + 2$), the following definitions will change:

$$\underline{A}_{(n)} = -\left(\underline{Y}_{BC(n)} + \alpha^{*} \underline{Y}_{CA(n)} + \alpha \underline{Y}_{AB(n)} \right), \quad (9)$$

$$\underline{\boldsymbol{I}}_{(n)} = \begin{bmatrix} \underline{Y}_{e(n)} + \underline{A}_{(n)} \\ \underline{Y}_{e(n)} + \alpha \cdot \underline{A}_{(n)} \\ \underline{Y}_{e(n)} + \alpha^{*} \cdot \underline{A}_{(n)} \end{bmatrix} \cdot \underline{\boldsymbol{U}}_{(n)}, \quad (10)$$

$$\underline{\boldsymbol{U}}_{(n)}^{\#} = \begin{bmatrix} \underline{U}_{A(n)} \\ \alpha \cdot \underline{U}_{B(n)} \\ \alpha^{*} \cdot \underline{U}_{C(n)} \end{bmatrix}, \quad \text{for} \quad n = 3k+2. \quad (11)$$

In order to present the formulas for all the harmonics in the correct and incorrect sequence, according to [1], the β complex rotation coefficient was introduced:

$$\beta = 1 e^{js\frac{2\pi}{3}}, \quad (12)$$

where:

$$s = \begin{cases} 1 & \text{for harmonics in correct sequence,} \quad n = 3k+1 \\ -1 & \text{for harmonics in incorrect sequence,} \quad n = 3k+2 \end{cases}$$

Using the β coefficient, it is possible to show unbalance admittation and a current-voltage equation, regardless of the phase sequence of harmonics (disregarding the harmonics of the zero sequence):

$$\underline{A}_{(n)} = -\left(\underline{Y}_{BC(n)} + \beta \underline{Y}_{CA(n)} + \beta^{*} \underline{Y}_{AB(n)} \right), \quad (13)$$

$$\underline{\boldsymbol{I}}_{(n)} = \begin{bmatrix} \underline{Y}_{e(n)} + \underline{A}_{(n)} \\ \underline{Y}_{e(n)} + \beta^{*} \cdot \underline{A}_{(n)} \\ \underline{Y}_{e(n)} + \beta \cdot \underline{A}_{(n)} \end{bmatrix} \cdot \underline{\boldsymbol{U}}_{(n)}. \quad (14)$$

Dependence (14) can be represented by a matrix equation of the form:

$$\underline{\boldsymbol{I}}_{(n)} = \underline{Y}_{e(n)} \cdot \underline{\boldsymbol{U}}_{(n)} + \underline{A}_{(n)} \cdot \boldsymbol{b} \cdot \underline{\boldsymbol{U}}_{(n)}, \quad (15)$$

where $\boldsymbol{b}$ is a rotation matrix equal to:

$$\boldsymbol{b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta^{*} & 0 \\ 0 & 0 & \beta \end{bmatrix}. \quad (16)$$

Using the Rotation Matrix (16), Equations (7) and (11) change to the form:

$$\underline{\boldsymbol{U}}_{(n)}^{\#} = \boldsymbol{b} \cdot \underline{\boldsymbol{U}}_{(n)}. \quad (17)$$

Thus, by substituting (17) to Equation (15), the following is obtained:

$$\underline{\boldsymbol{I}}_{(n)} = \underline{Y}_{e(n)} \cdot \underline{\boldsymbol{U}}_{(n)} + \underline{A}_{(n)} \cdot \underline{\boldsymbol{U}}_{(n)}^{\#}. \quad (18)$$

This means that for instantaneous values, the Current Vector (3) in the power line will be equal to:

$$\boldsymbol{i} = \sqrt{2} \cdot \Re e \sum_{n} \left\{ \left(G_{e(n)} + jB_{e(n)} \right) \cdot \underline{\boldsymbol{U}}_{(n)} + \underline{A}_{(n)} \cdot \underline{\boldsymbol{U}}_{(n)}^{\#} \right\} \cdot e^{jn\omega_{1}t}. \quad (19)$$

This vector can be separated into the sum of several components that interpret different physical phenomena:

$$\boldsymbol{i}_{c} = \sqrt{2} \Re e \sum_{n} G_{e(n)} \underline{\boldsymbol{U}}_{(n)} e^{jn\omega_{1}t} = \sum_{n} G_{e(n)} \boldsymbol{u}_{(n)}, \quad (20)$$

the reactive component:

$$\boldsymbol{i}_{r} = \sqrt{2} \Re e \sum_{n} jB_{e(n)} \underline{\boldsymbol{U}}_{(n)} e^{jn\omega_{1}t}, \quad (21)$$

the unbalance component:

$$\boldsymbol{i}_{u} = \sqrt{2} \Re e \sum_{n} \underline{A}_{(n)} \underline{\boldsymbol{U}}_{(n)}^{\#} e^{jn\omega_{1}t}, \quad (22)$$

and the active component of the current, which is consistent with the Fryze's theory, is:

$$\boldsymbol{i}_{a} = G_{e} \sum_{n} \boldsymbol{u}_{(n)}. \quad (23)$$

From the $\boldsymbol{i}_{c}$ component, it is possible to isolate the $\boldsymbol{i}_{s}$ scattering component:

$$\boldsymbol{i}_{s} = \boldsymbol{i}_{c} - \boldsymbol{i}_{a} = \sum_{n} G_{e(n)} \boldsymbol{u}_{(n)} - G_{e} \sum_{n} \boldsymbol{u}_{(n)} = \quad (24)$$

$$= \sum_n \left(G_{e(n)} - G_e \right) \cdot \boldsymbol{u}_{(n)} =$$
$$= \sqrt{2} \Re e \sum_n \left(G_{e(n)} - G_e \right) \cdot \underline{\boldsymbol{U}}_{(n)} e^{jn\omega_1 t} .$$

Thus, the Current Vector (19) can be decomposed into components:

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_u . \tag{25}$$

The effective values of the individual vector components are equal to:

$$\| \boldsymbol{i}_a \| = \frac{P}{\| \boldsymbol{u} \|} , \tag{26}$$

$$\| \boldsymbol{i}_s \| = \sqrt{ \sum_n \left(G_{e(n)} - G_e \right)^2 \| \boldsymbol{u}_{(n)} \|^2 } , \tag{27}$$

$$\| \boldsymbol{i}_r \| = \sqrt{ \sum_n B_{e(n)}^2 \| \boldsymbol{u}_{(n)} \|^2 } , \tag{28}$$

$$\| \boldsymbol{i}_u \| = \sqrt{ \sum_n A_{(n)}^2 \| \boldsymbol{u}_{(n)} \|^2 } . \tag{29}$$

The $\boldsymbol{i}_a$ active vector component of the current in accordance with the Fryze's theory depends on the active power of the receiver.

The $\boldsymbol{i}_s$ scattering vector component appears when the $G_{e(n)}$ equivalent conductivity of the receiver varies together with harmonics.

The $\boldsymbol{i}_r$ reactive vector component appears when there is a non-zero phase shift between the current vectors $\boldsymbol{i}_{(n)}$ and the voltage $\boldsymbol{u}_{(n)}$ for any harmonic in any phase. The condition for the existence of this shift is a non-zero equivalent susceptance $B_{e(n)} \neq 0$.

The $\boldsymbol{i}_u$ unbalance vector component appears in the case when the unbalance admittance $A_{(n)} \neq 0$.

The decomposition of the Current Vector (25) can be represented graphically as a parallel connection of several receivers interpreting different physical properties.

The presented graphical interpretation of three-phase current decomposition (Fig. 8) is a form that replaces a linear, stationary, unbalanced three-phase receiver for any n harmonic.

The components: $\boldsymbol{i}_a$ (active current), $\boldsymbol{i}_s$ (scatter), $\boldsymbol{i}_r$ (reactive) have the same sequence of harmonics as the supply voltage $\boldsymbol{u}$, while the unbalance $\boldsymbol{i}_u$ has the opposite sequence. For this reason, only the $\boldsymbol{i}_u$ unbalance current affects asymmetry of $\boldsymbol{i}$ currents.

Individual components are orthogonal, therefore they fulfill the following equation:

$$\| \boldsymbol{i} \|^2 = \| \boldsymbol{i}_a \|^2 + \| \boldsymbol{i}_s \|^2 + \| \boldsymbol{i}_r \|^2 + \| \boldsymbol{i}_u \|^2 . \tag{30}$$

The double-sided multiplication of this equation by the square of the effective value of the voltage vector $\| \boldsymbol{u} \|^2$ gives the power equation:

$$S^2 = P^2 + D_s^2 + Q^2 + D_u^2 . \tag{31}$$

where the individual components are:

- apparent power $S = \| \boldsymbol{u} \| \cdot \| \boldsymbol{i} \|$, [VA],
- active power $P = \| \boldsymbol{u} \| \cdot \| \boldsymbol{i}_a \|$, [W],
- reactive power $Q = \pm \| \boldsymbol{u} \| \cdot \| \boldsymbol{i}_r \|$, [VAr],
- unbalance power $D_u = \| \boldsymbol{u} \| \cdot \| \boldsymbol{i}_u \|$, [VA],
- scattering power $D_s = \| \boldsymbol{u} \| \cdot \| \boldsymbol{i}_s \|$, [VA].
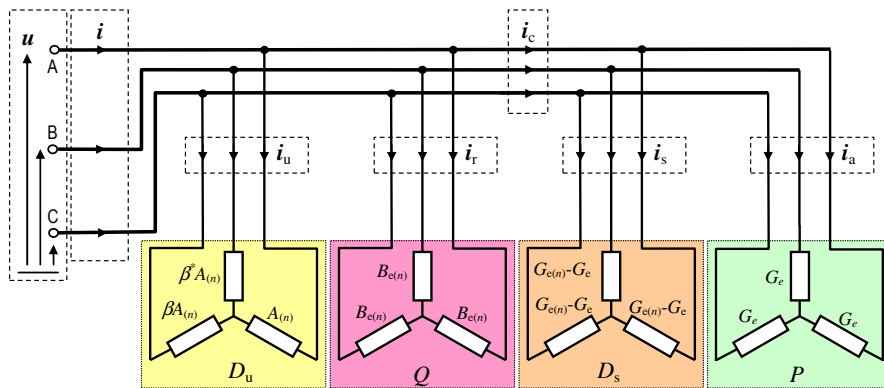

Fig. 8.   Graphical interpretation of the decomposition of three-phase AC components

The powers presented here do not have any physical interpretations. They are the result of multiplying the effective values of the physical components of the current with the voltage. Even the P active power does not have its physical interpretation, because in these analyzes it cannot take any negative values. This is only valid for passive receivers. For active receivers, one should pay attention to the sign, which depends on the current direction.

4. THE POWER COMPENSATION FOR HARMONIC WAVEFORMS FOR THREE PHASE CURRENT PHYSICAL COMPONENTS

In the case of three-phase three-wire circuits, with a balanced source supplying a balanced receiver, power compensation can be analyzed analogously to single-phase circuits, considered individually for each phase.

In a three-phase system, the sequence of harmonics must be controlled. The zero sequence (i.e. for $n = 3k + 3$) must be excluded from the analysis, as the CPC method does not provide for internal flows between phases for one frequency. When the source is symmetrical and the receiver is balanced, the appearance of these harmonics in the supply current is excluded, so the harmonics of the zero sequence need not be taken into account in the design process of the compensator.

The power factor is a parameter describing the efficiency of the entire current path, i.e. from the generation of energy, its transmission to use in the receiver. When decomposing the current using the CPC method, the power factor is equal to:

$$\lambda = \frac{P}{S} = \frac{\|\boldsymbol{i}_\mathrm{a}\|}{\sqrt{\|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{s}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}\|^2}} . \tag{32}$$

This coefficient is lowered not only by the $\boldsymbol{i}_\mathrm{r}$ reactive component, but also by the components of the $\boldsymbol{i}_\mathrm{s}$ scattering and $\boldsymbol{i}_\mathrm{u}$ unbalance.

The concept of power compensation using physical components of currents, in fact, is based on the determination of surrogate parameters of a three-phase receiver – equivalent susceptance $B_{\mathrm{e}(n)}$ and unbalance admittance $\underline{A}_{(n)}$ for subsequent harmonic frequencies. According to (32), the power factor is weakened by the $\boldsymbol{i}_\mathrm{s}$ scattering current, the $\boldsymbol{i}_\mathrm{u}$ unbalance current and the $\boldsymbol{i}_\mathrm{r}$ reactive current.

The reduction of $\boldsymbol{i}_\mathrm{r}$ reactive component in a three-phase system can be considered individually for each phase separately. The $\boldsymbol{i}_\mathrm{s}$ scattering current of the three-phase load cannot be compensated by a parallel reactance compensator. The method of compensation for this component is analogous to that for one phase.

By adding a parallel compensator (Fig. 9), it is not possible to reduce the $\boldsymbol{i}_\mathrm{s}$ component, so in this case the maximum power factor value according to Formulas (24) and (32) is:

$$\lambda = \frac{\|\boldsymbol{i}_\mathrm{a}\|}{\sqrt{\|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{s}\|^2}} < 1 . \tag{33}$$
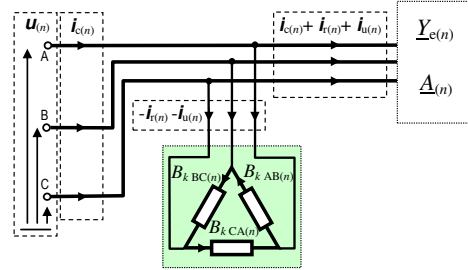
Fig. 9. A parallel compensator in a three-phase system for the nth harmonic

The $\boldsymbol{i}_\mathrm{u}$ unbalance current is compensated by decreasing the value of unbalance admittance. Formula (29) means that the receiver must be balanced for each harmonic separately.

The $\boldsymbol{i}_{\mathrm{r}(n)}$ reactive component is reduced when the imaginary part of the sum of the $\underline{Y}_{\mathrm{e}(n)}$ equivalent admittance and the compensator $\underline{Y}_{k(n)}$ is zeroed:

$$\begin{aligned}\Im m\{\underline{Y}_{\mathrm{e}(n)}\} + \Im m\{\underline{Y}_{k(n)}\} = \\ = B_{\mathrm{e}(n)} + \left(B_{k\,\mathrm{BC}(n)} + B_{k\,\mathrm{CA}(n)} + B_{k\,\mathrm{AB}(n)}\right) = 0.\end{aligned} \tag{34}$$

When the Unbalance Admittation (13) equals zero, the unbalance component $\boldsymbol{i}_{\mathrm{u}(n)}$ is reduced:

$$\underline{A}_{(n)} - j\left(B_{k\,\mathrm{BC}(n)} + \beta B_{k\,\mathrm{CA}(n)} + \beta^* B_{k\,\mathrm{AB}(n)}\right) = 0 . \tag{35}$$

The solution of Equations (34) and (35), taking into account the sequence of harmonics is the following dependence:

$$\begin{cases} B_{k\,\mathrm{AB}(n)} = \dfrac{1}{3}\left(s\sqrt{3}\Re e\{\underline{A}_{(n)}\} - \Im m\{\underline{A}_{(n)}\} - B_{\mathrm{e}(n)}\right) \\[2mm] B_{k\,\mathrm{BC}(n)} = \dfrac{1}{3}\left(2 \cdot \Im m\{\underline{A}_{(n)}\} - B_{\mathrm{e}(n)}\right) \\[2mm] B_{k\,\mathrm{CA}(n)} = \dfrac{1}{3}\left(-s\sqrt{3}\Re e\{\underline{A}_{(n)}\} - \Im m\{\underline{A}_{(n)}\} - B_{\mathrm{e}(n)}\right) \end{cases} . \tag{36}$$

The compensator, whose susceptance is determined from Dependence (36), resets the $\boldsymbol{i}_{\mathrm{r}(n)}$ reactive current and $\boldsymbol{i}_{\mathrm{u}(n)}$ unbalance for the nth harmonic.

Example

A single-phase receiver with $\underline{Z}_\mathrm{A} = (1 + j1)\,\Omega$ impedance is connected to a three-phase network.
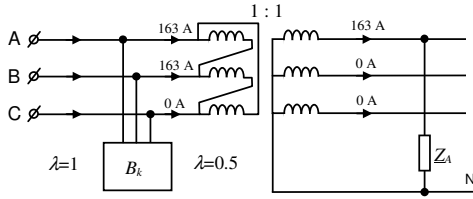
Fig. 10. Compensation of an unbalanced receiver

The circuit was powered by a lossless 1:1 transformer with a sinusoidal three-phase voltage with an effective value of $U = 230$ V. The phase current flowing in one phase is equal to:

$$I_A = \frac{U_A}{Z_A} = \frac{230}{\sqrt{1^2 + 1^2}} \cong 163 \text{ A}.$$

Phase admittance is:

$$\underline{Y}_{AB} = \frac{1}{\underline{Z}_A} = \left(\frac{1}{2} - j\frac{1}{2}\right)\text{S} \cong 0.71 e^{-j45^0}\text{S},$$

$$\underline{Y}_{BC} = 0,$$

$$\underline{Y}_{CA} = 0.$$

The equivalent admittance of the receiver accepts the following value:

$$\underline{Y}_e = G_e + jB_e = \underline{Y}_{AB} + \underline{Y}_{BC} + \underline{Y}_{CA} = \underline{Y}_{AB},$$

while the unbalance admittance is as follows:

$$\underline{A} = -\left(\underline{Y}_{BC} + \alpha \underline{Y}_{CA} + \alpha^* \underline{Y}_{AB}\right) =$$
$$= -\alpha^* \underline{Y}_{AB} = -1e^{-j120^0} \cdot 0.71 e^{-j45^0} =$$
$$= 0.71 e^{j15^0}\text{S} = (0.69 + j0.18)\text{S}.$$

The effective value of the voltage vector is:

$$\|\boldsymbol{u}\| = \sqrt{U_A^2 + U_B^2 + U_C^2} = \sqrt{3 \cdot 230^2} = 398 \text{ V}.$$

The effective values of currents, expressed by active, reactive and unbalanced components, are respectively:

$$\|\boldsymbol{i}_a\| = G_e \cdot \|\boldsymbol{u}\| = 0.5 \cdot 398 = 199 \text{ A},$$
$$\|\boldsymbol{i}_r\| = |B_e| \cdot \|\boldsymbol{u}\| = 0.5 \cdot 398 = 199 \text{ A},$$
$$\|\boldsymbol{i}_u\| = A \cdot \|\boldsymbol{u}\| = 0.71 \cdot 398 = 283 \text{ A}.$$

The total current is the sum of orthogonal components, thus:

$$\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2} = 399 \text{ A}.$$

The power factor of a three-phase receiver before compensation is:

$$\lambda = \frac{P}{S} = \frac{\|\boldsymbol{i}_a\|}{\|\boldsymbol{i}\|} = \frac{199}{399} = 0.5.$$

The compensator susceptance values from (36) are as follows:

$$B_{k\,AB} = \frac{1}{3}\left(\sqrt{3}\Re e\{\underline{A}\} - \Im m\{\underline{A}\} - B_e\right) =$$
$$= \frac{1}{3}\left(\sqrt{3} \cdot 0.69 - 0.18 + 0.5\right) = 0.51\,\text{S},$$

$$B_{k\,BC} = \frac{1}{3}\left(2 \cdot \Im m\{\underline{A}\} - B_e\right) =$$
$$= \frac{1}{3}\left(2 \cdot 0.18 + 0.5\right) = 0.29\,\text{S},$$

$$B_{k\,CA} = \frac{1}{3}\left(-\sqrt{3}\Re e\{\underline{A}\} - \Im m\{\underline{A}\} - B_e\right) =$$
$$= \frac{1}{3}\left(-\sqrt{3} \cdot 0.69 - 0.18 + 0.5\right) = -0.29\,\text{S}.$$

Positive susceptance values are realizable in a system with a capacitor, while a negative value is a system with inductance.
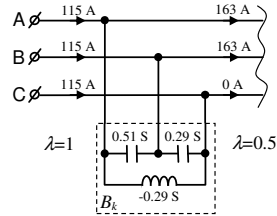

Fig. 11. Implementation of a compensator for this example

The applied compensator resets the reactive and unbalance components of current:

$$\|\boldsymbol{i}_a\| = 199 \text{ A},$$
$$\|\boldsymbol{i}_r\| = 0 \text{ A},$$
$$\|\boldsymbol{i}_u\| = 0 \text{ A}.$$

The effective value of the three-phase current has been reduced to: $\|\boldsymbol{i}\| = 199$ A. As a result of reactive power compensation and system balance, the power factor has been improved to $\lambda=1$. The effective values of the currents in the supply line in all phases are equal to:

$$\|\boldsymbol{i}_A\| = \frac{\|\boldsymbol{i}\|}{\sqrt{3}} \cong 115 \text{ A}.$$

5. THE ADAPTIVE POWER COMPENSATION

Susceptances B_k of the compensator (36) are the functions of: the B_e equivalent susceptance of the receiver, the $\underline{A}$ unbalance admittance and the s sequence of the harmonics. Therefore, after changing the parameters of the receiver, new settings of the compensator must be determined.
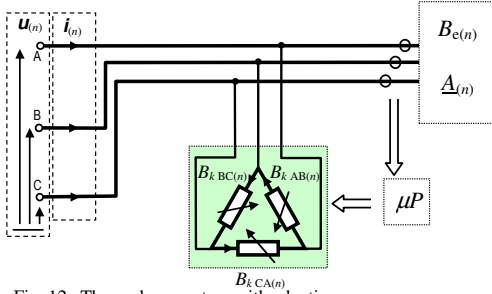

Fig. 12. Three-phase system with adaptive power compensation

The adaptive compensation (Fig. 12) will be based on the automatic determination of the compensator parameters depending on the instantaneous energy specifications of the receiver.

The system for adaptive power compensation is equipped with microprocessor systems (μP): measurement, identification and control, whose main idea of operation is the real-time determination of the compensator parameters based on the Dependence (36). The algorithm of this system assumes the realizability of the compensator for the border parameters of the receiver power changes. $\underline{S}_{AB}$, $\underline{S}_{BC}$, $\underline{S}_{CA}$ apparent powers of the individual phases of the receiver are values defined in a certain area of allowable power variations, around the mean value $\underline{S}_0 = P_0 + jQ_0$, which for all phases has the same value (Fig. 13). The area with the radius kS_0 is adopted to cover the apparent powers of all phases.
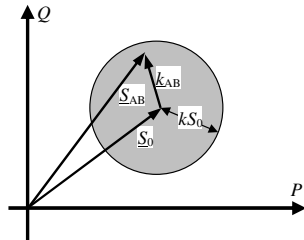

Fig. 13. The dispersion of the active and reactive power of the receiver for the AB phase

In the case of a receiver with a triangular topology with $\underline{Y}_{AB}$, $\underline{Y}_{BC}$, $\underline{Y}_{CA}$ admittance, the complex power of one of the phases is equal to:

$$\underline{S}_{AB} = P_{AB} + jQ_{AB} = \underline{U}_{AB} \cdot \underline{I}^*_{AB} = \underline{Y}^*_{AB} \cdot \|u_{AB}\|^2, \quad (37)$$

i.e. the admittance of one phase is equal to:

$$\underline{Y}_{AB} = \frac{\underline{S}^*_{AB}}{\|u_{AB}\|^2} . \quad (38)$$

The projection of the $\underline{S}_0$ reference vector (Fig. 13) is the $\underline{S}_{AB}$ vector shifted by a $\underline{k}_{AB}$ vector whose complex value is:

$$\underline{k}_{AB} = k_{AB} S_0 \cdot e^{j\vartheta_{AB}} , \quad (39)$$

where the k_{AB} coefficient is a positive value equal to or less than k. Complex power in the AB phase can therefore be expressed as the sum of:

$$\underline{S}_{AB} = \underline{S}_0 + \underline{k}_{AB} = \underline{S}_0 + k_{AB} S_0 \cdot e^{j\vartheta_{AB}} . \quad (40)$$

From Dependencies (38) and (40), it follows that admittance in this phase is:

$$\underline{Y}_{AB} = \frac{\underline{S}^*_0 + k_{AB} S_0 \cdot e^{-j\vartheta_{AB}}}{\|u_{AB}\|^2} = \\ = \underline{Y}_0 + k_{AB} Y_0 e^{-j\vartheta_{AB}}, \quad (41)$$

where $\underline{Y}_0 = \dfrac{\underline{S}^*_0}{\|u_{AB}\|^2} = Y_0 e^{-j\varphi_0}$.

The dispersion of the $\underline{Y}_{AB}$ admittance is possible within the accepted area, i.e. the borderline case is $k_{AB} = k$. The boundary parameters of the receiver, i.e. the equivalent susceptance and the unbalance admittance, determined from Dependencies (12) and (13) are respectively:

$$B_{e(n)} = \Im m\{\underline{Y}_{AB(n)} + \underline{Y}_{BC(n)} + \underline{Y}_{CA(n)}\} = \\ = -3Y_0 \sin\varphi_0, \quad (42)$$

$$\underline{A}_{(n)} = -\left(\underline{Y}_{BC(n)} + \beta\underline{Y}_{CA(n)} + \beta^*\underline{Y}_{AB(n)}\right) = \\ = -kY_0\left(e^{-j\vartheta_{BC(n)}} + \beta e^{-j\vartheta_{CA(n)}} + \beta^* e^{-j\vartheta_{AB(n)}}\right). \quad (43)$$

The susceptances of this compensator are determined from (36), (42) and (43) for each harmonic separately. The imaginary part of unbalance admittance for the correct and incorrect sequence of harmonics is:

$$\Im m\{\underline{A}_{(n)}\} = kY_0\big[\sin\vartheta_{BC(n)} + \\ + \sin(\vartheta_{CA(n)} - s120^0) + \sin(\vartheta_{AB(n)} + s120^0)\big] \quad (44)$$

where the s coefficient depends on the harmonic sequence and has been specified in Formula (12).

Due to the fact that none of the phases of the compensator is dominant, the limits of the compensator's susceptance for one of the phases must be calculated. The BC branch is the simplest mathematical form in Equality (36).

$$B_{k\,BC(n)} = \frac{1}{3}\left(2 \cdot \Im m\{\underline{A}_{(n)}\} - B_{e(n)}\right) =$$
$$= \frac{2}{3}kY_0\left[\sin\vartheta_{BC(n)} + \sin\left(\vartheta_{CA(n)} - s120^0\right) + \right.$$
$$\left. + \sin\left(\vartheta_{AB(n)} + s120^0\right)\right] + Y_0\sin\varphi_0. \tag{45}$$

The susceptances of this compensator can take positive and negative values, so the adaptive compensator topology must anticipate both situations. Physical implementation of such a compensator is possible in the arrangement of mechanical or electronic switches. Changing the reactive current of the coil is most often carried out by the serial connection of the T triac (Fig. 14).
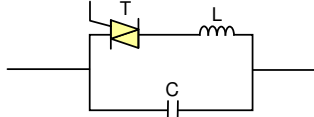

Fig. 14. Implementation of one phase of the adaptive compensator

The change of the susceptance of one phase of the compensator takes place by changing the ignition angle of the triac T. A positive susceptance value is available through the use of a capacitor C. The susceptance of one phase for such implementation is:

$$B_k = \omega_1 C - \frac{1}{\omega_1 L}\left(1 - \frac{2\alpha - \sin 2\alpha}{\pi}\right), \tag{46}$$

where α - firing angle of the triac.

Depending on the triac ignition angle, the minimum and maximum values of this susceptance are:

$$B_{k\,min} = \omega_1 C - \frac{1}{\omega_1 L},$$
$$B_{k\,max} = \omega_1 C. \tag{47}$$

The compensator with the adopted topology (Fig. 14) can generate additional current harmonics. They depend on the firing angle of the triac, with the third order being the largest of them. However, in the three-phase connection of the symmetrical compensator, the third harmonic current flows inside the compensator branch, without causing distortion of the compensating current. There may appear smaller contents of the odd order harmonics with the correct and incorrect sequence.

In the case of unbalanced operation of this compensator, the third harmonic causes a strong distortion of the current. This can be further compounded by the resonance between the compensator's capacity and the resultant induction of the source. For this reason, a compensator with such a topology cannot be used as a balancing compensator.

The system built according to [7] is devoid of this disadvantage:
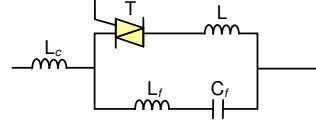

Fig. 15. Implementation of one phase of the adaptive compensator with the third harmonic filter

The modified compensator topology (Fig. 15) has been extended with a L_fC_f resonance filter whose resonant frequency is equal to the third harmonic. The additional L_c inductance helps to maintain the inductive character for harmonic frequencies. The individual parameters of the elements from Fig. 14 and Fig. 15 are determined from the equations for the minimum and maximum susceptance and from the resonant frequency of the filter.

6. CONCLUSIONS

A circuit constructed in accordance with Fig. 14 may cause a current deformation of 50%, whereas the modified circuit of Fig. 15 is characterized by a deformation of the order of 0.8%. The analysis presented concerns an adaptive compensator operating without feedback. It is also possible for such a compensator to work in a feedback system. In this case, measurements of currents and voltages (Fig. 12) should be made not on the receiver but on a three-phase source. The algorithm of such a compensator will control the trigger angle of the triac so that the measured current has a minimized reactive and unbalance component. The system working with feedback makes it necessary to check the stability of the work by a compensator algorithm.

Nomenclature

Symbols

$\underline{A}$	– complex number of unbalance admittance, S
α	– firing angle of the triac, °
α	– complex rotation coefficient
β	– complex rotation coefficient with the order of harmonics
$B_{e(n)}$	– equivalent susceptance of the receiver for the nth harmonic, S
B_k	– susceptance of compensator, S

b	–	rotation matrix
C	–	capacitance, F
D_s	–	scattering power, VA
D_u	–	unbalance power, VA
$G_{e(n)}$	–	equivalent conductance of the receiver for the nth harmonic, S
I_A	–	effective value of the current in phase A, A
$i_{A(n)}$	–	instantaneous current value on phase A for the nth harmonic, A
i	–	vector of the current, A
i_a	–	active vector component of the current, A
i_s	–	scatter vector component of the current, A
i_r	–	reactive vector component of the current, A
i_u	–	unbalance vector component of the current, A
L	–	inductance, H
λ	–	power factor, W/VA
n	–	harmonic number
P	–	active power, W
Q	–	reactive power, VAr
s	–	coefficient of harmonic sequence
S	–	apparent power, VA
T	–	period of the function, s
U_A	–	effective value of the voltage in phase A, V
$\|u\|$	–	resultant rms voltage for all harmonics, V
ω_1	–	basic pulsation, rad/s
$\underline{Y}_{AB}$	–	receiver admittance between phases A and B, S
$\underline{Y}_{e(n)}$	–	equivalent admittance of the receiver for the nth harmonic, S

Acronyms

AC	–	Alternating Current
CCS	–	Combined Charging System
CPC	–	Currents' Physical Components
DC	–	Direct Current
EV	–	Electric Vehicle
μP	–	Microprocessor Systems

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Konrad Zajkowski received his M.Sc. degree in Electronics, specialty: Automatics and Robotics from Koszalin University of Technology (1998). In 2008, he received a Ph.D in Electrical Engineering at Szczecin University of Technology. Since 2004, he has been a research worker at the Department of Electrical Engineering at the Koszalin University of Technology, where he currently works as a senior lecturer. His scientific interests focus on issues related to power theory, overvoltages and ways of limiting them, and modern methods control of electric motors. He has participated in 1 international and 2 national research projects, presenting the results of his work at international and national conferences, published over 100 scientific articles in international and national magazines, book chapters as well as conference materials. He is also the author of two monographs, three national patents and six academic books.