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CHARACTERISTICS OF STRESS DISTRIBUTION OF A MULTI LAYERED CYLINDRICAL PRESSURE VESSEL

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Abstract: The operation of a multilayer pressure vessel subjected to thermomechanical loads is very significant. The cylindrical pressure vessel is widely used in industrial engineering: for example, to hold a variety of different types of liquid. On thick-walled cylinders, various loading circumstances such as internal overpressure, external overpressure, heat, bending, twisting, and combinations of these load characteristics are applied. Researchers have developed a number of strategies for enhancing the strength of cylinders, including the use of multilayer cylinders and increasing the thickness of the walls. This paper presents the results of an analytical and numerical analysis of a three-layer cylinder. The Abaqus FEA software is used to determine temperature, displacement, and stress distribution of a multilayer cylinder considering the effect of centripetal and centrifugal heat flow. From the numerical analysis, it is observed that centrifugal heat flux is more hazardous than centripetal heat flux for a multi-layered cylinder under thermo-mechanical loading.

Keywords: stress distribution, thermo-mechanical loads, multilayer pressure vessel, centripetal and centrifugal, analytical and numerical analysis

1. INTRODUCTION

A thick-walled pressure vessel operating at very high temperatures and pressure is one of the most important structures in modern industries. The use of composite and functionally graded materials is increasing as these materials are highly capable of controlling stress due to thermal and mechanical loading. A more practical case can be observed in an engine cylinder where the temperature and pressure is not fixed and it changes with time. Additionally, thicker walls can increase the pressure capacity of thick cylinders by increasing the thickness and employing shrink fits, where multilayers are shrunk together with different diametric differences to create compound cylinders. Typically, while working with a three-layer compound cylinder, the outer layer 3 is contracted on to the intermediate layer 2 first, followed by the resulting compound pressure vessel being contracted on to the inner layer 1. It is easily visible that as the outer layer contracts as a result of cooling, the inner layer is compressed and the outer layer is similarly compressed, resulting in a state of tension between the two layers of the structure. Whenever this compound pressure vessel is subjected to mechanical loading, the resultant hoop stresses will be the algebraic sum of the stresses arising from the internal pressure and the stresses resulting from shrinkage, which is the case here [1]. Numerous researchers have conducted various kinds of analyses to increase the conformity of the cylinder during operation. The elasto-plastic technique was used to investigate the behavior of thick-walled cylinders under mechanical loading [2]. Tajana Vasko explored the continuity of the stress and displacement field at the interface of a compound cylinder in the elastic plastic range [3]. A highly efficient and easily applicable analytical solution was derived for heated and pressurized cylinders by Sollund *et al.* [4]. A cylinder subjected to thermomechanical loading was examined using the finite element method to determine the stress and displacement field [5]. Zhang *et al.* derived the thermo-mechanical stress of a multilayered cylinder considering the effect of closed-end analytically [6]. Vedeld *et al.* developed an expression for two-layer cylinders exposed to pressure and thermal gradient for investigating stress and displacement induced in the body [7]. Thermal loadings have a significant impact on stress distribution in the case of a pressure vessel analysis. Laminated cylinders were investigated numerically by S. Aksoy *et al.* [8]. A residual stress analysis was carried out for a long cylinder by Y. V. Tokovyy and C. C. Ma [9]. Simulation of a multilayer pressure vessel was carried out by Ameya Palekar *et al.* [10]. Siva Krishna Raparla analysed a multilayer high pressure vessel numerically [11]. An optimization of a three-layer pressure vessel was carried out by Ossama R. Abdelsalam [12]. Anwar Kandil investigated a pressure vessel under constant and cyclic loading both analytically and numerically [13]. A design of a pressure vessel and its numerical analysis was carried out by I. Satyanarayana and K. Praveena [14]. A stress analysis of a gun barrel subjected to dynamic loading was investigated by H. Babaie [15]. A structural analysis of a thick-walled cylinder was investigated by Pial das and Md. Shahidul Islam using FEM [16]. An analytical method was developed and a numerical analysis was presented to investigate composite cylinder under various loading by Elgohary *et al.* [17].

2. MATHEMATICAL MODELING

A steady state heat equation in cylindrical coordinate can be expressed as:

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)T(r)=0. \tag{1}$$

T(r) represents the temperature distribution along the radial axis. Thus, the temperature distribution function of an arbitrary layer is denoted by:

$$T_K(r)=\frac{T_K-T_{K-1}}{ln\, q_k}ln\frac{r}{r_k}+T_K, \tag{2}$$

where T_K between the k_{th} and $k+1_{th}$ layer can be determined by:

$$T_K=\frac{\lambda_K T_{K-1}\, ln\, q_{k+1}+\lambda_{K+1}T_{K+1}\, ln\, q_k}{\lambda_K\, ln\, q_{k+1}+\lambda_{K+1}\, ln\, q_k}. \tag{3}$$

The interfacial radius, thermal conductivity, and temperature are all represented in this equation by, *R*, *T*, and *k* (1, 2, *n*). The k_{th} layer is indicated by the subscript *k*, and the radius ratio, *q*, is given by the equation $q_k=\frac{r_k}{r_{k-1}}$.

Considering thermal expansion, the Hooks material law for plane stress is given by [18]:

$$\begin{bmatrix}\sigma_{rr,i}\\ \sigma_{\theta\theta,i}\end{bmatrix}=\frac{E_i}{1-\nu^2}\begin{bmatrix}1 & \nu_i\\ \nu_i & 1\end{bmatrix}\begin{bmatrix}\xi_{rr,i}-\alpha_i\Delta T_i\\ \xi_{\theta\theta,i}-\alpha_i\Delta T_i\end{bmatrix}. \tag{4}$$

The continuity of radial stress and displacement is summarized in the following mathematical formula [19]:

$$\begin{gathered}{\sigma_r}^1(b)=\sigma_r^2(b); u_1(b)=u_2(b),\\ {\sigma_r}^2(c)=\sigma_r^3(c); u_2(c)=u_3(c)\ .\end{gathered} \tag{5}$$

The equation to calculate hoop stress is given by [20]:

$$\sigma_\theta=-P_o+\frac{P_i-P_o}{F-1} \tag{6}$$

where: $F=C_1\times C_2\times C_3\ldots\times C_r\times C_{r+1}\ldots\times C_n$

C_{r+1} is given by $C_{r+1}=\frac{2{K^2}_{r+1}}{1+{K^2}_{r+1}}$ and $K_{r+1}=\frac{d_{r+1}}{d_r}$

3. FINITE ELEMENT MODEL

Layer 1 has an inner radius of 10 mm and an outer radius of 20 mm. The cylinder is in the middle, with an inner radius of *b* of 20 mm and an outer radius of *c* of 30 mm. In the outer cylinder, the inner radius is 30 mm, while the external radius is *d* of 40 mm. It is possible to find bimetallic interfaces when the radius of the bimetallic interface is 20 mm and when the radius of the bimetallic interface is 30 mm. Figure 1 shows the mesh of the model.

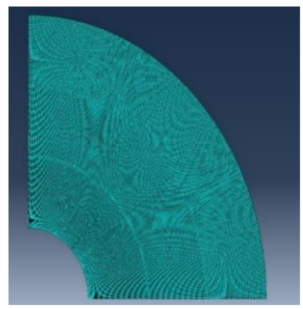

Fig. 1. Mesh of the model

Choosing the CPE4T mesh element type shows that the element has four nodes and can handle biquadratic displacement, bilinear temperature, and decreased integration.

4. MATERIAL PROPERTIES

Three different materials, i.e. aluminum, titanium and steel are used in this work. For layer 1, aluminum is used, layer 2 is composed of titanium and layer 3 is made of steel. Characteristics of the materials were given in Table 1.

Tab. 1. The characteristics of the materials that were used in the analysis.

Materials	*E* (GPa)	ϑ	α (°C)×10^{-6}	K (W m^{-1} K^{-1})
Aluminium 1050A-H9 (layer 1)	72	0.33	24	234
Titanium (layer 2)	108	0.3	11	20
Steel ASTM A564 H1150 (layer 3)	210	0.3	11.6	19.5

5. RESULT ANALYSIS

Three-layered pressure vessel problem was solved with the help of the ABAQUS program in this research project. Here, two different cases were analyzed, as shown on Figure 2.

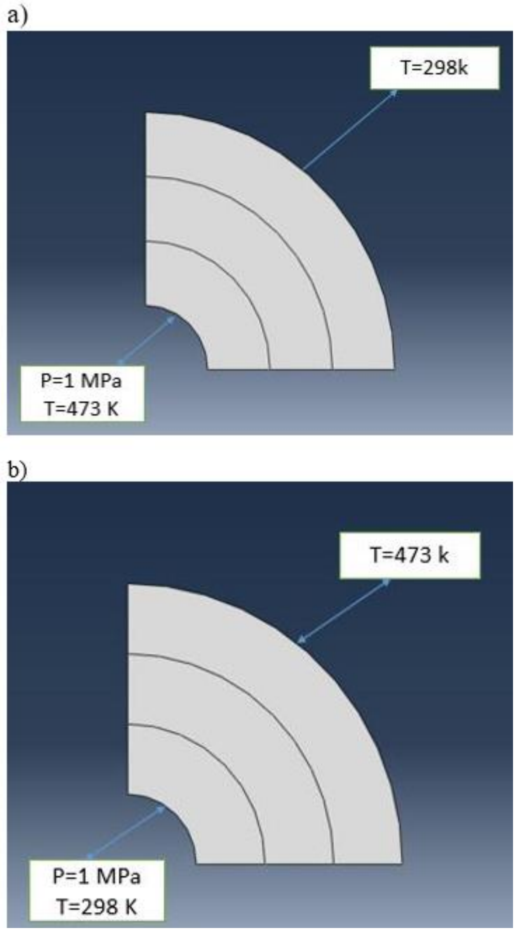

Fig. 2. Model showing boundary condition: a) case I; b) case II of the analysis

In Figure 2, two different boundary conditions can be noticed. Firstly, hot fluid passes through the cylinder, and then cold fluid passes through the cylinder.

1. Case I: When hot fluid passes through the cylinder, the temperature inside the cylinder is greater than the outside of the cylinder. Hence, heat will flow from the inner surface to the outer surface, i.e. the centrifugal flux.
2. Case II: When cold fluid passes through the cylinder, the temperature outside the cylinder is greater than the inside of the cylinder. Hence heat will flow from the outer surface to the inner surface i.e. the centripetal flux.

Figure 3 and 4 show comparison of analysis for temperature along the radius and for distribution of radial stress along the radius respectively.

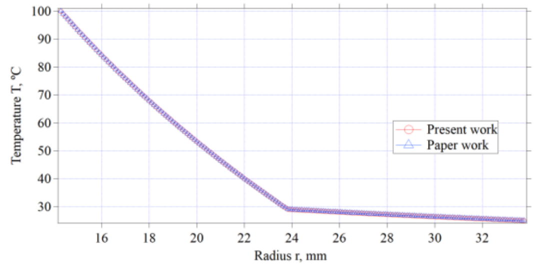

Fig. 3. Comparison of the present analysis with reference paper [19] analysis for the variation of temperature along the radius

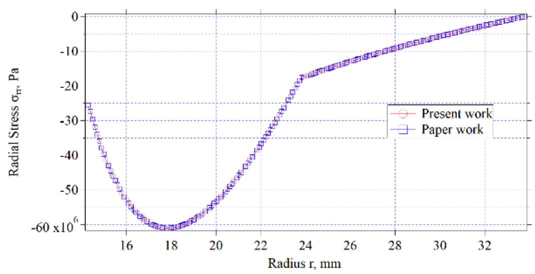

Fig. 4. Comparison of the present analysis with reference paper [19] analysis for the distribution of radial stress along the radius

For the verification of this research work, the result obtained from running the commercial code using Abaqus FEA software is compared to the result of Kaoutar Bahoum *et al.* [19]. The results achieved in the present analysis are in good agreement with the reference paper where the highest error percentage is less than 1%.

From the mesh sensitivity analysis (Fig. 5), the results obtained using 117000 elements are similar to the results obtained using 144189 elements.

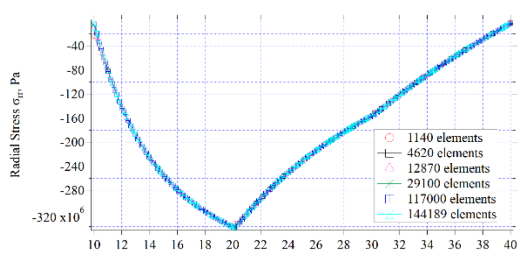

Fig. 5. Mesh sensitivity analysis for the current model

To save the time of calculations, the optimum number of elements for further analysis is 117000 elements.

Figure 6 indicates the temperature distribution through the radial distance imposed on thermo-mechanical loads.

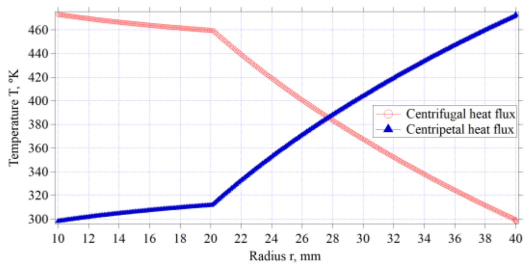

Fig. 6. Variation of temperature through radius imposed to thermo-mechanical loads

In the case of the centrifugal heat flux, temperature is higher in the inside region and, for the centripetal heat flux, temperature is higher in the outside surface, thus satisfying boundary conditions. There is a sharp change of temperature at the first interface at r = 20 mm, but the temperature change is not so sharp at the second interface at r = 30 mm. The temperature distribution is continuous throughout the regions.

In Figure 7, it can be observed that the maximum radial stress due to the centrifugal thermal flux is greater than that of the centripetal flux.

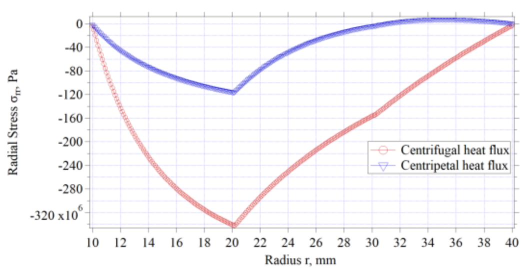

Fig. 7. Distribution of radial stress through radius imposed on thermo-mechanical loads

Thus, the centrifugal thermal flux is more dangerous than the centripetal thermal flux. A sharp change of radial stress occurs at the first interface at r = 20 mm. The radial stress distribution is continuous throughout the regions and thus satisfies the boundary condition.

The variation of hoop stress changes when subjected to thermo-mechanical loading is shown in Figure 8.

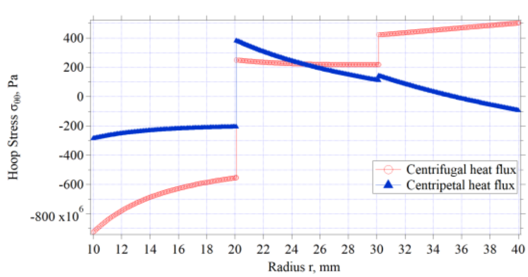

Fig. 8. Temperature-dependent variation of hoop stress as a function of radius applied to thermo-mechanical stresses

The maximum hoop stress exists at the outer surface. Discontinuity occurs in hoop stress distribution at the interfaces. At the first interface, r = 20 mm discontinuity is large but at the second interface, r = 30 mm discontinuity is slightly smaller. The discontinuity occurs due to the variation of the elastic properties of the materials. It is found that the maximum thermo-mechanical stress due to the centrifugal thermal flux is greater than that of the centripetal flux. Therefore, the centrifugal thermal flux poses a greater threat than the centripetal thermal flux.

Radial displacement is caused by a temperature gradient as well as mechanical loading on the system. In Figure 9, it is observed that the maximum radial displacement due to the centrifugal thermal flux is greater than that of the centripetal flux.

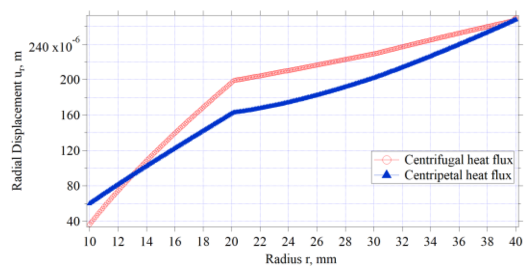

Fig. 9. Thermo-mechanical loads are subjected to radial displacement variations with increasing radius

Thus, the centrifugal thermal flux is more dangerous than the centripetal thermal flux. A sharp change of radial displacement occurs at the first interface at r = 20 mm. Radial displacement distribution is continuous throughout the regions and thus it satisfies the boundary condition.

Figure 10 indicates that discontinuity occurs in Von Mises stress distribution at the interface.

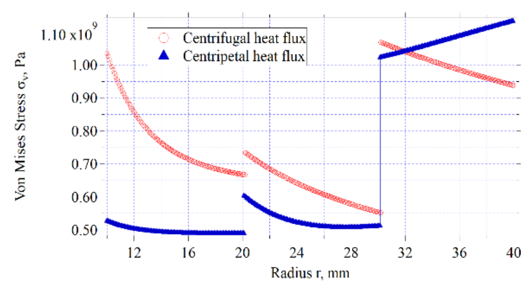

Fig. 10. Variation of Von Mises stress through radius subjected to thermo-mechanical loadings

It is observed that the maximum stress at the interface due to the centrifugal thermal flux is greater than that of the centripetal flux. As a result, the centrifugal thermal flux is riskier than the centripetal thermal flux.

6. CONCLUSIONS

In this study, the variation of displacement, temperature and stress across the radial distance exposed thermo-mechanical loads is investigated. The following conclusions are drawn from the numerical analysis:

1. In order to assure the reliability of the cylinder, it is necessary to study the effects of both pressure and temperature.
2. In comparison to the mechanical load, the hoop stress component is more sensitive to thermal loads.
3. Temperature, displacement and stress distribution are continuous near the interface region.
4. The centrifugal heat flux is more threatening than the centripetal heat flux.

Nomenclature

Symbols

σ_{rr} – radial stress, Pa
$\sigma_{\theta\theta}$ – hoop stress, Pa
E – young modulus, GPa
ϑ – poisson ratio
T – temperature, K
K – thermal Conductivity, $W\,m^{-1}\,K^{-1}$
R – interfacial Radius, m
α – thermal Expansion coefficient, (℃)
u – displacement, m
P – internal Pressure, MPa
P_0 – external Pressure, MPa
K_{r+1} – ratio of outer diameter to inner diameter

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Biographical notes

Somnath Somadder received his M.Sc. degree in mechanical engineering from Khulna University of Engineering & Technology in 2021 (specialization: Computational Solid Mechanics). Currently, he is working as an Assistant Professor in the Department of Mechanical Engineering, Khulna University of Engineering & Technology. He has published 8 scientific papers in international journals and conference proceedings. His scientific interest is focused on Multiscale modelling, composite materials, analysis of stress singularity fields in 3D transversely isotropic piezoelectric and elastic bonded joints using the finite element method.

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